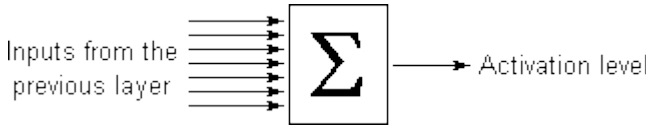
Web Site of Robert John Morton

Neural Networks: MLP Input Summation Function

Outline, *precision*, *speed*, *loop optimisation*, *testing*, *conclusion*, *final solution*.

Outline

Each neuron in a multi-layer perceptron network receives a number of inputs. A neuron's summation function multiplies each of its inputs by a corresponding weight to form a weighted input. It then sums all the weighted inputs to produce a single output which is called the neuron's activation level:



This activation level for a 7-input neuron is calculated according to the formula:

$$\text{Activation Level} = \frac{1}{7} * \sum_{i=0}^{i=6} \text{input}_i * \text{weight}_i$$

The activation level is fed as the input to the neuron's Sigmoid Function to produce the neuron's output. See document file sigmoid.htm.

Precision

To make our neural network as fast as possible, we will restrict all inputs and outputs to the bipolar range of a 16-bit integer, ±32767. We will also restrict the weights to this range. Suppose our neuron has 7 inputs as shown above. The full process for summing the 7 inputs and producing an activation-level output therefore will be:

$$\text{Activation Level} = \frac{1}{7 * 32768} * \sum_{i=0}^{i=6} \text{input}_i * \text{weight}_i$$

Two 16-bit integers produce a 32-bit product. Furthermore, the sum of the 32-bit products for a large number of inputs could be a number requiring far more than 32-bits. Suppose the inputs to our neuron are stored in an array **int I[]** where **int** is a 16-bit signed quantity. Suppose also that the corresponding weights are to be found in an array **int W[]**. The easy way out in computing the weighted sum while preserving the original precision would be therefore to use a floating-point double variable as the sum accumulator as follows:

```
int i;
double x;
for(x = 0, i = 0, i < 7; i++)
x += (double)*(I + i) * *(W + i);
x /= 7 * 32768;
```

The integer input value *(I + i) must be typecast to a floating-point double before the multiply takes place so that the product is floating-point double precision. The weight will be typecast to a double automatically to match the forced typecasting of the input value. To scale the final output back to the input range of ±32767, we must, once the loop has terminated, divide by 32768 and by the number of inputs. This final value of the accumulator is then typecast automatically to a 16-bit signed integer when passed as input to the neuron's **int Sigmoid( )** function.

Speed

Floating-point arithmetic is easy and trouble-free. But it is also very slow compared with integer arithmetic.

> [In this document we shall assume that our neural network program will be running on a PC with an Intel 80286 16-bit CPU. The coding can easily be adapted to take advantage of longer word-lengths.]

We will find out the best form of 'C' coding for a fast integer version of our neural input summation program by investigating various methods as follows.

1) All-Integer Method

We will first try to compute the sum of the neuron's weighted inputs as follows:

```
int i, x;
for(x = 0, i = 0, i < 7; i++)
  x += *(I + i) * *(W + i);
x /= 7;
```

However, this truncates the 32-bit product immediately to 16-bit precision. Furthermore, the accumulator x could overflow with as few as two inputs.

2) Long Accumulator

We can prevent overflow in the accumulator x by defining it as a **long** (32-bit) variable. The summation loop would therefore become:

```
int i;
long x;
for(x = 0, i = 0, i < 7; i++)
  x += *(I + i) * *(W + i);
x /= 7 * 32768;
```

Notice that the final result, because it is 32-bits, has to be divided by 32768 as well as by the number of inputs in order to yield the final 16-bit 'activation level' we want. This final division could be made faster using a right-shift of 15:

```
x = (x >> 15) / 7;
```

Unfortunately, since we are adding together truncated products and then dividing the accumulated sum by the number of inputs, we are losing half the possible 32-bit precision of each input * weight product. The result will therefore not be a true 16-bit representation of the sum of the weighted inputs. We need to create and take account of the full 32-bit product.

3) Long Product

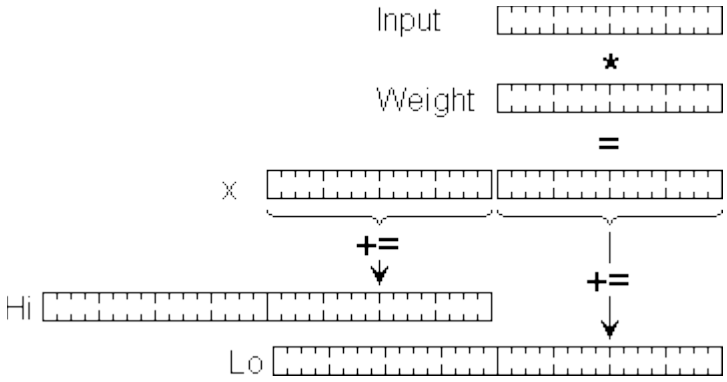
We can preserve the 32-bit precision of the product by typecasting one of the operands to a long as follows:

```
int i;
long x;
for(x = 0, i = 0, i < 7; i++)
  x = (long)*(I + i) * *(W + i);
x /= 7;
```

Since we typecast the input to a long, the weight is automatically typecast to a long before the multiplication is performed. We therefore get a 32-bit long product. However, we cannot simply add this long product to a long accumulator. It could overflow after only two inputs.

4) Split Long Accumulator

A 48-bit accumulator on the other hand could accommodate 32-bit input * weight products from 32k inputs. This is more than we are likely to need. We can effectively create a 48-bit accumulator by using two separate 32-bit **long** accumulators as follows:



We add the top half of the 32-bit product to the Hi accumulator and the bottom half to the Lo accumulator. We can write this in 'C' as follows:

```
int i, AL;
long x, Lo, Hi;
for(Hi = 0, Lo = 0, i = 0, i < 7; i++) {
  x = (long)*(I + i) * *(W + i);
  Hi += x >> 16;
  Lo += x & 0xFFFF;
}
AL = ((Hi << 1) + (Lo >> 15)) / 7;
```

Once the loop has terminated, the lower half of Lo is discarded. The upper half of Lo is added into Hi. Be aware that because x is right-shifted by 16 before it is added to Hi, Hi holds the sum of the products already divided by 65536. We therefore multiply it by 2 (left-shift it by 1). It has thus been effectively divided by 32768. We then divide Lo by 32768 by right-shifting it by 15. We then add the two results together and divide by the number of inputs. The long result is then assigned to the int variable AL.

This indeed works perfectly with only a binary rounding error on the least significant bit of the resulting 16-bit activation level, AL. Furthermore, it performs its task in one thirty-seventh of the time taken by the equivalent floating-point double computation.

5) In-line Assembler

Although it makes a small part of a program specific on a particular type of CPU chip, in-line assembler can in some cases add extra vital speed at the speed-critical heart of a program. For the small amount of re-coding needed to migrate to another processor, the speed advantage of in-line assembler can sometimes be worth it. But not always.

When multiplying two integers on the Intel 80x86 family of processors, you put one operand in the AX register and multiply by the other operand. The 32-bit product then appears in the two registers DX and AX. In theory this saves the overhead of typecasting (in effect both operands) to **long** before the multiplication is performed. It is coded as follows:

```
int i, ni, w, AL;   //AL = Activation Level
unsigned int q;
long    Lo, Hi;
for(Hi = 0, Lo = 0, i = 0, i < 7; i++) {
  ni = *(I + i);    //neural input
  w = *(W + i);     //weight
  _asm {
    mov ax, ni      ;load input into AX register
    imul w          ;signed multiply with weight
    mov ni, dx      ;put ms word into ni
    mov q, ax       ;put ls word into unsigned q
  }
  Hi += ni;         //add ms word into High accum
  Lo += q;          //add ls word into Low  accum
}
AL = ((Hi << 1) + (Lo >> 15)) / 7;
```

Note that ordinary 'C' int variables can be used within in-line assembler.

The neural input ni is placed in the AX register and a signed multiply is then performed with the weight w.

The most-significant (the upper) half of the product is then copied into ni. Integer ni is used here because at this point it has finished serving its primary purpose of carrying the neural input, and using it for the upper half of the product saves defining a separate variable for this purpose.

The least-significant (the lower) half of the product is copied into q. This is an *unsigned* integer so that the bits in AX are copied literally without any complementing. This is because the arithmetic sign of the whole product is effectively conveyed within the upper half of the product in ni.

For all this, testing showed the in-line assembler version of the weighted input summation loop to be 41.6% slower than the split accumulator 'C' version. We must bear in mind that the compiler has to push registers on to the stack before dropping into in-line assembler and popping them off again afterwards. This extra chore obviously outweighs the small saving gained by being able to lift the two halves of the double-precision product directly from the DX and AX registers.

Loop Optimisation

The final way of making sure the code was as fast as possible was to try a different method of accessing the inputs and weights. Instead of adding the index i to the start addresses of the inputs and weights arrays, two pointers pi and pw were defined to point at the current input and current weight respectively and incremented each pass of the loop. Two versions were tried. The first increments the pointers in the **for** statement:

```
int i, AL, *pi = I, *pw = W;
long x, Lo = 0, Hi = 0;
for(i = 0, i < 7; i++, pi++, pw++) {
  x = (long) *pi * *pw;
  Hi += x >> 16;
  Lo += x & 0xFFFF;
}
AL = ((Hi << 1) + (Lo >> 15)) / 7;
```

The second version increments the pointers in the multiply statement:

```
int i, AL, *pi = I, *pw = W;
long x, Lo, Hi;
for(Hi = 0, Lo = 0, i = 0, i < 7; i++) {
  x = (long) *pi++ * *pw++;
  Hi += x >> 16;
  Lo += x & 0xFFFF;
}
AL = ((Hi << 1) + (Lo >> 15)) / 7;
```

Both were found to be 4.6% slower than the original so were abandoned.

Testing

Before deciding on Method 4 above as the optimum way of implementing the weighted inputs summation function, all the above methods were fully tested and exercised. Three types of test were performed. Each was done by embedding the code for each method in an exerciser program.

Test 1

Using the floating-point double computation as a benchmark, each of 77 inputs was multiplied by a corresponding weight, and the products were accumulated. The final sum was divided by the number of inputs. The results were then written to a file called results.txt. Below is the exerciser for Test 1 with the embedded code fragments to be tested shown in bold. This source code is in the file wsum1.c:

```
#include <stdio.h>
#define NI 77    //number of inputs to the neuron
int I[] = {      //inputs
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329
};

int W[] = {      //weights
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
```

```
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557
};

int i;
char s[512];
FILE *fh;

double Benchmark(void) { //floating-point double as benchmark
  int i;
  double x;
  for(x = 0, i = 0; i < NI; i++)
    x += (double)*(I + i) * *(W + i);
  return(x / (NI * 32768));
}

int integer(void) {      //all-integer method
  int i, x;
  for (x = 0, i = 0; i < NI; i++)
    x += *(I + i) * *(W + i);
  return(x / NI);
}

int LongAccum(void) {    //int product with long accumulator
  int i;
  long x;
  for(x = 0, i = 0; i < NI; i++)
    x += *(I + i) * *(W + i);
  return(x / NI);
}

int LongProd(void) {     //long product with long accumulator
  int i;
  long x;
  for(x = 0, i = 0; i < NI; i++)
    x += (long)*(I + i) * *(W + i);
  return((x >> 15) / NI);
}

int SplitLong(void) {   //long product with 2 long accumulators
  int i;
  long x, Hi, Lo;
  for(Hi = 0, Lo = 0, i = 0; i < NI; i++) {
    x = (long)*(I + i) * *(W + i);
    Hi += x >> 16;      //add upper half of long product to Hi
    Lo += x & 0xFFFF;   //add lower half of long product to Lo
  }
  return(((Hi << 1) + (Lo >> 15)) / NI);
}

int SplitASM(void) {    //in-line assembler version of above
  int i, ni, w;
  unsigned int q;
  long Hi, Lo;
```

```
  for(Hi = 0, Lo = 0, i = 0; i < NI; i++) {
    ni = *(I + i);      //current input amplitude
    w = *(W + i);       //corresponding input weight
    _asm {
      mov ax, ni        ;load integer input into AX register
      imul w            ;do signed mpy with integer weight
      mov ni, dx        ;put ms word into unsigned integer p
      mov q, ax         ;put ls word into unsigned variable q
    }
    Hi += ni;           //add ms word into high accumulator
    Lo += q;            //add ls word into low accumulator
  }
  return(((Hi << 1) + (Lo >> 15)) / NI);
}

void Test(int flag) {
  int c, AL1, AL2, AL3, AL4, AL5;
  char *r;
  double AL0;
  AL0 = Benchmark();  //double computation as benchmark
  AL1 = integer();    //all-integer method
  AL2 = LongAccum();  //integer product with long accumulator
  AL3 = LongProd();   //long product with long accumulator
  AL4 = SplitLong();  //long product with 2 long accumulators
  AL5 = SplitASM();   //in-line assembler version of the above
  i += sprintf(s + i,
    "   Benchmark using floating-point double      %+9.2f\n"
    , AL0);
  i += sprintf(s + i,
    "   1) int  products with  int accumulator     %+6d  \n"
    , AL1);
  i += sprintf(s + i,
    "   2) int  products with long accumulator     %+6d  \n"
    , AL2);
  i += sprintf(s + i,
    "   3) long products with long accumulator     %+6d  \n"
    , AL3);
  i += sprintf(s + i,
    "   4) long products with 2 long accumulators  %+6d  \n"
    , AL4);
  i += sprintf(s + i,
    "   5) In-line assembler version of the above  %+6d\n\n"
    , AL5);
  if(flag) sprintf(s + i,"\n");
  for(r = s; (c = *r) > '\0'; r++)
    putc(c, fh);                        //write it to file
}

main() {
  fh = fopen("results.txt","a");    //open file for appending
  i = sprintf(s,
    "TEST RESULTS FOR THE 5 COMPUTATION METHODS\n\n"
  );
  i += sprintf(s + i,"For positive inputs...\n");
  Test(0);                          //test for positive inputs
  for(i = 0; i < NI; i++)           //reverse inputs
    I[i] = -I[i];
  i  = sprintf(s, "For negative inputs...\n");
  Test(1);                          //test for negative inputs
```

```
  fclose(fh);                       //close the file
}
```

The results produced in the file results.txt by running the above exerciser (wsum1.exe) are as follows:

```
TEST RESULTS FOR THE 5 COMPUTATION METHODS

For positive inputs...
   Benchmark using floating-point double      +13485.75
   1) int  products with  int accumulator       +189
   2) int  products with long accumulator      −8321
   3) long products with long accumulator       −132
   4) long products with 2 long accumulators  +13485
   5) In-line assembler version of the above  +13485
For negative inputs...
   Benchmark using floating-point double      −13485.75
   1) int  products with  int accumulator       −189
   2) int  products with long accumulator      +8321
   3) long products with long accumulator       +132
   4) long products with 2 long accumulators  −13485
   5) In-line assembler version of the above  −13485
```

Clearly, methods 1, 2 and 3 do not work. Register overflow has obviously occurred. Methods 4 and 5 clearly work, but notice that the integer values have been effectively truncated at the least-significant bit: not rounded. The truncation occurs symmetrically for both positive and negative values. This solution is perfectly acceptable.

Test 2

Again using the floating-point double computation as a benchmark, the two methods that worked properly were used to find the weighted sum of the 77 inputs. The time taken by each method to perform the summation 10,000 times was measured. A variant of Method 4 using incremented pointers *pi and *pw was also timed. The results were then appended to the results.txt file.

Following is the exerciser for Test 2 with the embedded code fragments to be tested shown in bold. This source code is in the file wsum2.c:

```
#include<stdio.h>
#include<sys\types.h>
#include<sys\timeb.h>
#define NI 77                   //number of inputs to the neuron

int I[] = {                     //inputs
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329,
  11376, 13425, 17920, 30226, 28763, 18940, 15329
};
```

```
int W[] = {                   //weights
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557,
  12345, 21345, 31245, 16730, 31662, 25460, 13557
};

double GetTime() {
  struct timeb TimeData;
  ftime(&TimeData);
  return((double)(TimeData.millitm) / 1000 + TimeData.time);
}

double Dfun() {
  int k, i;
  double x, t = GetTime();
  for(k = 0; k < 10000; k++) {
    for(x = 0, i = 0; i < NI; i++)
      x += (double)(*(I + i)) * *(W + i);
    x /= NI * 32768;
  }
  return(GetTime() - t);
}

double Lfun1(void) {
  int k, i, a;
  long x, Hi, Lo;
  double t = GetTime();
  for(k = 0; k < 10000; k++) {
    for(Hi = 0, Lo = 0, i = 0; i < NI; i++) {
      x = (long)*(I + i) * *(W + i);
      Lo += x & 0xFFFF;
      Hi += x >> 16;
    }
    a = ((Hi << 1) + (Lo >> 15)) / NI;
  }
  return(GetTime() - t);
}

double Lfun2(void) {
  int k, i, a, *pi = I, *pw = W;
  long x, Hi, Lo;
  double t = GetTime();
  for(k = 0; k < 10000; k++) {
    for(Hi = 0, Lo = 0, i = 0; i < NI; i++) {
      x = (long)*(pi+ i) * *(pw + i);
      Lo += x & 0xFFFF;
      Hi += x >> 16;
    }
    a = ((Hi << 1) + (Lo >> 15)) / NI;
  }
  return(GetTime() - t);
}
```

```
double Lfun3(void) {
  int k, i, a, *pi = I, *pw = W;
  long x, Hi, Lo;
  double t = GetTime();
  for(k = 0; k < 10000; k++)
    for(Hi = 0, Lo = 0, i = 0; i < NI; i++) {
      x = (long)*pi++ * *pw++;
      Lo += x & 0xFFFF;
      Hi += x >> 16;
    }
    a = ((Hi << 1) + (Lo >> 15)) / NI;
  }
  return(GetTime() - t);
}

double Lfun4(void) {
  int k, i, a, *pi = I, *pw = W;
  long x, Hi, Lo;
  double t = GetTime();
  for (k = 0; k < 10000; k++) {
    for (Hi = 0, Lo = 0, i = 0; i < NI; i++, pi++, pw++) {
      x = (long)*pi * *pw;
      Lo += x & 0xFFFF;
      Hi += x >> 16;
    }
    a = ((Hi << 1) + (Lo >> 15)) / NI;
  }
  return(GetTime() - t);
}

double Afun(void) {
  int k, i, AL, ni, w;
  unsigned int q;
  long Hi, Lo;
  double t = GetTime();
  for (k = 0; k < 10000; k++) {
    for (Hi = 0, Lo = 0, i = 0; i < NI; i++) {
      ni = *(I + i);
      w  = *(W + i);
      _asm {
        mov ax, ni    ;load integer input into AX register
        imul w        ;do signed multiply with integer weight
        mov ni, dx    ;put ms word into unsigned integer p
        mov q, ax     ;put ls word into unsigned variable q
      }
      Hi += ni;       //add ms word into High accumulator
      Lo += q;        //add ls word into Low accumulator
    }
    AL = ((Hi << 1) + (Lo >> 15)) / NI;
  }
  return(GetTime() - t);
}
```

```
main() {
  FILE *fh;
  char s[1024], *p = s;
  long o;
  int c, i;
  double T1, T2, T3, T4, T5, T6;
  for(T1 = GetTime() + 10; GetTime() < T1;); //wait 10 seconds
  printf("\n\nTiming...\n");
  T1 = Dfun(); T2 = Lfun1(); T3 = Lfun2();
  T4 = Lfun3(); T5 = Lfun4(); T6 = Afun();

  i  = sprintf(s,"TIME TRIALS AGAINST FLOATING-POINT \
DOUBLE BENCHMARK:\n");
  i += sprintf(s + i,"   Floating-point double  %6.2f \
secs  (Benchmark)          \n", T1);
  i += sprintf(s + i,"   4) 'C' using *(I+i)    %6.2f \
secs  (%6.2f times as fast)\n", T2, T1/T2);
  i += sprintf(s + i, "   4) 'C' using *(pi+i)   %6.2f \
secs  (%6.2f times as fast)\n", T3, T1/T3);
  i += sprintf(s + i,"   4) 'C' using *pi++     %6.2f \
secs  (%6.2f times as fast)\n", T4, T1/T4);
  i += sprintf(s + i,"   4) 'C' for(..., pw++)  %6.2f \
secs  (%6.2f times as fast)\n", T5, T1/T5);
  i += sprintf(s + i,"   5) asm using *(I+i)    %6.2f \
secs  (%6.2f times as fast)\n", T6, T1/T6);
  sprintf(s + i,"Note: time trials were only done on the \
methods that worked.\n\n\n");

  fh = fopen("results.txt","a");     //open for appending
  for (p = s; (c = *p) > '\0'; p++)  //write it to file
    putc(c, fh);
  fclose(fh);                        //close the file
}
```

The results produced in the file results.txt by running the above exerciser (wsum2.exe) are as follows:

```
TIME TRIALS AGAINST FLOATING-POINT DOUBLE BENCHMARK:
Floating-point double  345.15 secs  (Benchmark)
4) 'C' using *(I+i)      9.34 secs  ( 36.95 times as fast)
4) 'C' using *(pi+i)     9.39 secs  ( 36.76 times as fast)
4) 'C' using *pi++       9.72 secs  ( 35.51 times as fast)
4) 'C' for(..., pw++)    9.61 secs  ( 35.92 times as fast)
5) asm using *(I+i)     15.99 secs  ( 21.59 times as fast)
Note: time trials were only done on the methods that worked.
```

Test 3

This was to check that the function behaved properly at the extremities of its numeric range. The test was done only on Method 4. With all weights set to 32767, the computation was again done on all 77 inputs and their corresponding weights, first with all inputs set to +32767, then with all inputs set to zero, then with all inputs set to −32767. The results were then further appended to the results.txt file.

Below is the exerciser for Test 3 with the embedded code fragment to be tested shown in bold. This source code is in the file wsum3.c:

```
#include <stdio.h>
#define NI 77//number of inputs to the neuron
int I, W = 32767;
long SplitLong(void) { //long product with 2 long accumulators
  int i;
  long x, Hi, Lo;
  for(Hi = 0, Lo = 0, i = 0; i < NI; i++) {
    x = (long)I * W;
    Hi += x >> 16;
    Lo += x & 0xFFFF;
  }
  return(((Hi << 1) + (Lo >> 15)) / NI);
}

main() {
  int i, c;
  long x;
  char s[512], *r;
  FILE *fh;
  fh = fopen("results.txt","a");     //open file for appending
  i  = sprintf(s    ,
    "TESTING WITH MAXIMUM INPUT AND WEIGHT\n"
  );
  I =  32767; x = SplitLong(); c = x;
  i += sprintf(s + i,
    "   For max positive inputs  %+6d  %9lX\n", c, x
  );
  I =  0; x = SplitLong(); c = x;
  i += sprintf(s + i,
    "   For zero inputs...       %+6d  %9lX\n", c, x
  );
  I = -32767; x = SplitLong(); c = x;
  i += sprintf(s + i,
    "   For max negative inputs  %+6d  %9lX\n\n\n", c, x
  );
  for (r = s; (c = *r) > '\0'; r++)
    putc(c, fh);
  fclose(fh);                        //close the file
}
```

The results produced in the file results.txt by running the above exerciser (wsum3.exe) are as follows:

```
TESTING WITH MAXIMUM INPUT AND WEIGHT
For max positive inputs  +32766   00007FFE
For zero inputs...           +0   00000000
For max negative inputs  -32766   FFFF8002
```

Notice that the output of the summation can never reach 32767. However, this could only become a problem in a network comprising an impracticably large number of neural layers. If it does become a problem, arrangements will have to be made for setting weights to 32768 which would overflow a 16-bit integer by one bit.

Conclusion

The best coding for the weighted inputs summation function is as follows:

```
int i, al;
long x, Hi, Lo;
for (Hi = 0, Lo = 0, i = 0; i < NI; i++) {
  x = (long)*(I + i) * *(W + i);
  Hi += x >> 16;      //add upper half of long product to Hi
  Lo += x & 0xFFFF;   //add lower half of long product to Lo
}
al = ((Hi << 1) + (Lo >> 15)) / NI;
```

where NI is the externally-defined number of inputs, I is the start address of the integer inputs array, W is the start address of the integer weights array, al is the resulting neural activation level. This code performs the task at 37 times the speed of a floating-point double computation.

Final Solution

When implemented within a neural network, the base addresses of the input and weight arrays are unlikely to be available directly as constants I and W as they were in the exerciser programs. These addresses will probably be passed to the summation function in pointer-variables, say, *pi and *pw. Finally, therefore, the above coding was adapted as follows:

```
int i, al, *pi, *pw;
long x, Hi, Lo;
for(pi = I, pw = W, Hi = 0, Lo = 0, i = 0; i < NI; i++) {
  x = (long)*(pi + i) * *(pw + i);
  Hi += x >> 16;      //add upper half of long product to Hi
  Lo += x & 0xFFFF;   //add lower half of long product to Lo
}
al = ((Hi << 1) + (Lo >> 15)) / NI;
```

This was then added to the time trials exerciser and was found to be only half a percent slower than using I and W directly.

This is therefore the code that was recommended for use in the neuron's weighted input summation function.
